

# Ginzburg-Landau theory of an RVB superconductor

V. N. Muthukumar<sup>a</sup> and Z. Y. Weng<sup>b</sup>

<sup>a</sup> Department of Physics, Princeton University, Princeton, NJ 08544

<sup>b</sup> Center for Advanced Study, Tsinghua University, Beijing 100084, China

We present a Ginzburg-Landau formulation of the *bosonic* resonating-valence-bond (RVB) theory of superconductivity. The superconducting order parameter is characterized by phase vortices that describe *spinon* excitations and the transition to the superconducting state occurs when such phase vortices (un)bind. We show that the boson RVB theory always leads to  $hc/2e$  flux quanta, and that the presence of a trapped spin-1/2 moment inside a vortex core gives rise to observable consequences for the low temperature field-dependent specific heat. We also show that the cores of magnetic fluxoids exhibit enhanced antiferromagnetic correlations.

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Following the original proposal of Anderson [1], there is, by now, a considerable body of literature devoted to the study of the resonating-valence-bond (RVB) theory. Much of this is based on the so called  $t - J$  model, which is the simplest possible model describing electrons (holes) moving in an antiferromagnetic background [2]. The RVB theories postulate that such a system is best described by collective spin and charge degrees of freedom (usually called spinons and holons in the literature), rather than a quasiparticle theory of interacting electrons. While there is no satisfactory proof that the quasiparticle theory fails in the  $t - J$  or related models, there are some indications for such a failure, both from theoretical and numerical studies. Phenomenology of the cuprate superconductors based on these ideas has also been fairly successful. Motivated by these considerations, we present, in this paper, an effective theory of a superconducting RVB state. The theory leads to several interesting experimental consequences, and is very different from the conventional BCS theory of superconductivity. Since the theory makes definite predictions, its veracity can be tested easily.

The theory we present, is based on a bosonic description of the  $t - J$  model. We choose this description since it accounts very well for the short-range antiferromagnetic (AF) correlations that play an important role in the cuprate superconductors. Further, in the limit of zero doping, the  $t - J$  model reduces to the Heisenberg model and the bosonic RVB theory in this limit (Schwinger-boson mean field theory) provides an excellent description of the AF long-range ordered ground state, and the excited states [3], [4]. In this limit, the bosonic RVB state can also be related to the variational wave function of Liang, Doucot and Anderson [5], as shown by Chen [6]. Away from half filling, the theory describes *bosonic* charge (holon) and spin (spinon) degrees of freedom. In the bosonic theory of the RVB state [7], the electron operator is expressed in terms of the bosonic holon and spinon operators and a *topological (vortex) phase operator* as

$$c_{i\sigma} = h_i^\dagger b_{i\sigma} e^{i\hat{\Theta}_{i\sigma}}. \quad (1)$$

It satisfies usual fermionic anticommutation relations. The phase operator  $\hat{\Theta}_{i\sigma}$  is the most important ingredient of the theory. It arises because the doped holes move in a spin background (not necessarily ordered) with AF correlations, and reflects the nonlocal effects of adding a hole to a doped Mott AF insulator. In what follows, we shall be concerned with the holon and spinon degrees of freedom, and therefore, it is natural to ask what role does the phase operator play in the description of these degrees of freedom. As shown in [7], rewriting the  $t - J$  model in terms of the bosonic representation (1) leads to the emergence of two link fields,  $A^s$  and  $A^h$ . The link field  $A^s$  is coupled to the *holon* degrees of freedom and describes fictitious fluxoids bound to *spinons* satisfying  $\sum_c A_{ij}^s = \pm\pi \sum_{l \in c} \sigma n_{ls}^b$ , for an arbitrary closed path  $c$ . Here,  $n_\sigma^b$  denotes the spinon number operator. Similarly, the link field  $A_{ij}^h$  is coupled to the *spinon* degrees of freedom and describes fictitious fluxoids bound to *holons* satisfying  $\sum_c A_{ij}^h = \pm\pi \sum_{l \in c} n_l^h$ , where  $n^h$  is the holon number operator. Thus, we are led to the following physical picture. The motion of holes in an AF background leads to nonlocal correlations between the charge and spin degrees of freedom, that are described by the link fields. The holons feel the presence of the spinons as vortices (quantized flux tubes) and *vice versa* (see Fig. 1). Inasmuch the holons and spinons perceive their mutual presence through the  $\pi$  flux quanta, this theory can also be thought of as a  $\pi$  flux theory, albeit one where the  $\pi$  fluxoids are *bound to the constituent particles*.

The effective Hamiltonian is given by  $H_{\text{eff}} = H_h + H_s$ , where  $H_h$  and  $H_s$  are the holon and spinon Hamiltonians respectively. Let us begin with the holon Hamiltonian,  $H_h$ . For convenience, we work with a continuum model and write

$$H_h \approx \frac{1}{2m_h} \int d^2\mathbf{r} \, h^\dagger(\mathbf{r}) (-i\nabla - \mathbf{A}^e - \mathbf{A}^s)^2 h(\mathbf{r}), \quad (2)$$

where  $m_h \simeq (2t_h a^2)^{-1}$  ( $t_h \sim t$ ) is the effective mass of the holon, and  $\mathbf{A}^e$ , the vector potential of the external electromagnetic field. Note that the holon Hamiltonian (2) is coupled to the spinons through the term,  $\mathbf{A}^s(\mathbf{r})$ .

This term is the continuum version of the link field  $A_{ij}^s$  and is given by

$$\mathbf{A}^s(\mathbf{r}) = \frac{1}{2} \int d^2\mathbf{r}' \frac{\hat{\mathbf{z}} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} [n_\uparrow^b(\mathbf{r}') - n_\downarrow^b(\mathbf{r}')] . \quad (3)$$

The spinon Hamiltonian  $H_s$  is given by

$$H_s \approx -\frac{J}{2} \sum_{\langle ij \rangle \sigma} \left[ \Delta_{ij}^s \left( e^{i\sigma A_{ij}^h} \right) b_{i\sigma}^\dagger b_{j-\sigma}^\dagger + \text{h.c.} \right] , \quad (4)$$

where the spinon pairing (RVB) order parameter is defined by  $\Delta_{ij}^s = \sum_\sigma \langle e^{-i\sigma A_{ij}^h} b_{i\sigma} b_{j-\sigma} \rangle$ . Note again, the presence of the *holon* link variables in the definition of the *spinon* pairing (RVB) order parameter. For zero doping, the order parameter is identical to that of the Schwinger-boson mean field theory. As shown in [7], the Hamiltonian (4) can be solved for a fixed hole concentration, within a self-consistent mean field theory. The bosonic spinon excitations are gapped, and the gap vanishes as the hole doping decreases to zero. The short-range AF correlations are determined by the RVB order parameter  $\Delta_{ij}^s$ .

When the holons (Bose) condense, the system should become superconducting, since the spinons are already paired. However, the presence of the phase field  $\hat{\Theta}_{i\sigma}$  in (1) leads to interesting consequences. To see this, let us first write down the superconducting order parameter in terms of the decomposition (1). Assuming singlet pairing of electrons on nearest neighbor sites, we get (in the continuum limit),

$$\Delta_{\hat{\eta}}(\mathbf{r}) = \Delta_{\hat{\eta}}^0 e^{i\Phi^s(\mathbf{r})} , \quad (5)$$

where  $\Delta_{\hat{\eta}}^0 = f_{\hat{\eta}} \Delta^s [\psi_h^*]^2$  ( $f_{\hat{\eta}} = \pm 1$  for  $\hat{\eta} = \hat{x}, \hat{y}$ ),  $\psi_h^*(\mathbf{r}) = \langle h^\dagger(\mathbf{r}) \rangle$  denotes the Bose condensate of the holons, and  $\Phi^s(\mathbf{r})$  arising from the phase field  $\hat{\Theta}_{i\sigma}$  in (1) is given by

$$\Phi^s(\mathbf{r}) = \int d^2\mathbf{r}' \text{Im} \ln [z - z'] (n_\uparrow^b(\mathbf{r}') - n_\downarrow^b(\mathbf{r}')) , \quad (6)$$

where  $z = x + iy$ . From (5) and (6), it is clear that  $\Phi^s$  describes phase vortices centered around the spinons:  $\Phi^s \rightarrow \Phi^s \pm 2\pi$ , if the coordinate  $\mathbf{r}$  winds around a spinon continuously in space. Evidently,  $\Phi^s$  is related to  $\mathbf{A}^s$  ( $\nabla \Phi^s = 2\mathbf{A}^s$ ) and both describe the vortex effect related to the spinon excitations. From (2) and (5), we see that a generalized Ginzburg-Landau (GL) equation can be written down for the holon condensate  $\psi_h(\mathbf{r})$  as

$$\alpha \psi_h + \beta |\psi_h|^2 \psi_h + \frac{1}{2m_h} (-i\nabla - \mathbf{A}^e - \mathbf{A}^s)^2 \psi_h = 0 . \quad (7)$$

The current operator can be constructed in the usual manner as

$$\mathbf{J}(\mathbf{r}) = -\frac{i}{2m_h} \left[ \psi_h^\dagger(\mathbf{r}) \nabla \psi_h(\mathbf{r}) - \nabla \psi_h^\dagger(\mathbf{r}) \psi_h(\mathbf{r}) \right] - \frac{\mathbf{A}^e + \mathbf{A}^s}{m_h} \psi_h^\dagger(\mathbf{r}) \psi_h(\mathbf{r}) . \quad (8)$$

At  $T = 0$  and in the absence of an external magnetic field, all the spinons are (RVB) paired. Then, we see from (3) that  $\mathbf{A}^s$  is trivial and the phase of the superconducting condensate is robust. But *excited* spinons can (and do) influence the holon condensate through the link field  $\mathbf{A}^s(\mathbf{r})$  in the above GL equation. In the following, we shall investigate such effects systematically.

*Superconductivity, confinement, and  $T_c$ :* From the definition of the order parameter (5), we infer that the presence of a holon condensate is *not* sufficient for superconductivity to occur; *i.e., phase coherence will not be realized in the system if the spinon vortices unbind*, such that  $\langle e^{i\Phi^s(\mathbf{r})} e^{-i\Phi^s(\mathbf{r}')} \rangle$  falls off exponentially. This leads us to define the transition temperature,  $T_c$ , as the temperature at which the spinon vortices bind, resulting in the vanishing of  $\Phi^s$ . To relate  $T_c$  to the number of excited spinons, let us first consider the energy it costs to create an isolated spinon vortex. For a single spinon vortex centered around the origin, we have  $\mathbf{A}^s(\mathbf{r}) = \frac{1}{2} \frac{\hat{\mathbf{z}} \times \mathbf{r}}{r^2}$ , for distances  $r \gg a_c$ , the size of the vortex core. From (8), we see that the presence of a spinon creates a supercurrent in the holon condensate through  $\mathbf{A}^s$  [see also (11) below]:  $\mathbf{J} = -\frac{\rho_h}{m_h} \mathbf{A}^s$ , as illustrated by Fig. 2(a). Substituting the above in (2), we get

$$E_v = - \int d^2\mathbf{r} \mathbf{A}^s \cdot \mathbf{J} - \int d^2\mathbf{r} \rho_h \frac{(\mathbf{A}^s)^2}{2m_h} = \frac{\rho_h}{2m_h} \int d^2\mathbf{r} (\mathbf{A}^s)^2 \quad (9)$$

$$= \frac{\pi \rho_h}{4m_h} \int dr \frac{1}{r} \propto \ln \frac{L}{a_c} , \quad (10)$$

where  $L$  is the size of the sample. In general, the cost of creating more than one spinon vortex can be obtained by substituting the expression for  $\mathbf{A}^s(\mathbf{r})$ , *viz.*, (3) in (2). It is easily seen that the resulting expression describes a logarithmic attractive (repulsive) interaction between vortex-antivortex (vortex) excitations, like in the conventional Berezinskii-Kosterlitz-Thouless (BKT) transition.

From the above, we conclude that a single  $S = 1/2$  bosonic spinon excitation is forbidden (owing to a logarithmically diverging energy) and all excited spinons should be bound in vortex-antivortex pairs like in the low temperature phase of a BKT system. Consequently,  $\Phi^s$  in (6) becomes trivial and phase coherence of (5) is realized. The ground state is therefore a superconductor in which  $S = 1/2$  bosonic spinon excitations are confined by a logarithmic potential. However, such a confinement potential does *not* preclude the existence of fermionic quasi-particle excitations. Elsewhere, we showed that  $S = 1/2$

nodal quasiparticles can be created from the condensate as a composite excitations. A calculation of the spectral function in the superconducting RVB state shows the presence of quasiparticle peaks below  $T_c$ , consistent with results from angle-resolved photoemission spectroscopy [8]. Furthermore, an  $S = 1$  spin excitation can be constructed from a pair of bosonic  $S = 1/2$  spinons, as shown in Fig. 2(b). The excitation energy  $E_g$ , in this case, is finite and can be determined from the bosonic mean field theory [7]. This excitation leads to a sharp feature in the dynamical spin correlation function below  $T_c$ . Note that the vorticities of the current vortices are not directly related to the spin polarization directions, due to a gauge freedom to be discussed below.

The transition to the normal state occurs when the phase coherence of the order parameter is destroyed; *i.e.*,  $\Phi^s$  in (5) is disordered owing to the emergence of free spinon vortices and the deconfinement of bosonic spinons marks the transition. It should be noted that the temperature at which the unbinding of spinon vortex-antivortex excitations takes place can be substantially lower than the conventional BKT transition temperature ( $\sim 1000$  K at optimal doping if  $\mathbf{A}^s$  is neglected [9]). This is because the cores of the spinon vortices begin to touch each other before the unbinding of vortices-antivortices driven by entropy happens. In this dense limit, the vortex-antivortex excitations can unbind because the energy cost is no longer logarithmically divergent, and  $T_c$  is the temperature at which the average distance between spinons  $l \simeq 2a_c$ , where  $a_c$  is the core radius of a spinon vortex, as illustrated in Fig. 3. The latter can be estimated by solving the GL equation for an isolated vortex [10]. Here, we merely point out that the spinons have a characteristic length scale since their dynamics is governed by a uniform flux of  $\delta\pi$  per plaquette. Recall that the spinons perceive the presence of holons as  $\pi$  fluxoids. The gauge field  $A_{ij}^h$  in (4) represents an average flux of  $\delta\pi$  (per plaquette) when the holons are condensed and the spinons acquire a characteristic length scale,  $a_c \sim a/\sqrt{\pi\delta}$ . Now, if  $n_s^{ex}$  be the number of excited spinons, then,  $l = 2a/\sqrt{\pi n_s^{ex}}$ . Therefore, we conclude that  $T_c$  is determined by the condition  $n_s^{ex} = \delta$ . The RVB mean field theory of (4) shows that this condition is satisfied at a temperature  $T_c \sim E_g/4$ ,  $E_g$  being the characteristic spinon energy [7], as plotted in Fig. 3, where the experimental data from YBCO [11] are also shown for comparison (taking  $J = 100$  meV). Such an estimate of  $T_c$  based on the “core touching” mechanism agrees very well with the results from a systematic renormalization group analysis of (2) by M. Shaw *et al.* [12].

*Meissner effect and flux quantization:* We now consider the situation when an external magnetic field is present at  $T = 0$ . Writing  $\psi_h(\mathbf{r}) = \sqrt{\rho_h} e^{i\phi_h(\mathbf{r})}$ , we express the supercurrent given by (8) as

$$\mathbf{J} = \frac{\rho_h}{m_h} [\nabla\phi_h - \mathbf{A}^e - \mathbf{A}^s] . \quad (11)$$

Following the usual arguments for single valuedness of  $\psi_h(\mathbf{r})$ , we get

$$\frac{m_h}{\rho_h} \oint_c \mathbf{J}(\mathbf{r}) \cdot d\mathbf{r} = 2\pi n - \oint_c d\mathbf{r} \cdot (\mathbf{A}^e + \mathbf{A}^s) , \quad (12)$$

where the integral is over a closed loop and  $n$ , an integer. Now suppose that the integration is carried over a loop that is far away from the core of the vortex. Then,  $\mathbf{J} = 0$  along the loop and we get

$$\left( 2\pi n - \oint_c d\mathbf{r} \cdot \mathbf{A}^e \right) - \oint_c d\mathbf{r} \cdot \mathbf{A}^s = 0 . \quad (13)$$

When  $\mathbf{A}^s = 0$ , we see that the magnetic flux is quantized at  $2\pi n$  in units of  $\hbar c/e$ ; *i.e.*, the minimal flux quantum in this case is  $\hbar c/e \equiv \Phi_0$ , as expected for a charge  $e$  Bose system. However, the presence of  $\mathbf{A}^s$  changes the quantization condition radically. Suppose there is one excited spinon trapped in the core of a magnetic fluxoid [Fig. 4(a)]. Then, from (13), we obtain the minimal flux quantization condition,

$$\oint_c d\mathbf{r} \cdot \mathbf{A}^e = \pm\pi , \quad (14)$$

which is precisely the quantization condition of  $\Phi_0/2$  in a superconductor with  $2e$  pairing. As the holons do not distinguish between internal (fictitious) and external (magnetic) flux in (2), they still perceive a total flux quantized at  $\Phi_0$  [see Fig. 4(a)], even though the true magnetic flux quantum is  $\Phi_0/2$ .

*Stability of the  $2e$  flux quantum:* When the magnetic flux quantum inside the core is  $\Phi_0$ , there is no spinon trapped inside the core. On the other hand, a bosonic spinon is trapped inside the core if the flux quantum is  $\Phi_0/2$ . Therefore, we have to estimate the energy difference between these two cases to determine which of these is energetically favorable. The energy difference (per unit length)  $\Delta\epsilon$  between  $\Phi_0/2$  and  $\Phi_0$  magnetic fluxes due to the magnetic field and supercurrent is given by  $\Delta\epsilon = -3 \left( \frac{\Phi_0}{8\pi\lambda} \right)^2 \ln \kappa$  where  $\lambda = (m_h c^2 / 4\pi e^2 \delta)^{1/2}$ ,  $\kappa = \lambda/a_c$ , and  $a_c \sim 1/\delta^{1/2}$ , as discussed earlier. Then,  $\Delta\epsilon = -\frac{3\pi}{8} t_h \delta \ln \kappa \geq -t\delta$ , which vanishes as  $\delta \rightarrow 0$ . So far, we have ignored the cost of the vortex core. Since the  $\Phi_0/2$  flux has a spinon trapped inside the core, we need to add the energy cost of an excited spinon to  $\Delta\epsilon$  estimated above. Since the energy of an excited spinon in the bulk (within the boson RVB theory [7]) is  $E_s = E_g/2 \sim J\delta$ , and since  $t > J$ , we expect the  $\Phi_0/2$  flux quantum to be favorable for all doping concentrations [13]. Furthermore, as will be argued below, spinon (RVB) pairing inside the core is actually *improved* over the bulk; *viz.*, the cost of creating a spinon inside the core is smaller than the energy of an excited spinon in the bulk. This further ensures the stability of the  $\Phi_0/2$  flux quantum. This result

has to be contrasted with the slave boson RVB theories, where a  $\Phi_0$  flux quantum can be stabilized for small doping [14], [15]. In a recent paper [16], Wynn *et al.*, carried out a careful search for  $\Phi_0$  fluxoids in the high temperature superconductors. Using scanning SQUID and Hall probe studies, they looked for  $\Phi_0$  fluxoids in underdoped YBCO, but only observed  $\Phi_0/2$  fluxoids [16]. If spin charge separation indeed occurs in the high temperature superconductors, these experiments support our finding that only  $\Phi_0/2$  fluxoids are favored energetically.

*Local moment inside a vortex core:* As discussed above, each magnetic flux  $hc/2e$  is associated with an unpaired spin  $S = 1/2$  trapped inside the vortex core (Fig. 4), where the holon density is suppressed. Note that the flux quantization condition (14) is independent of the spin polarization. While  $\oint_c d\mathbf{r} \cdot \mathbf{A}^s = \pm\pi$  does depend on the spinon polarization [cf.(3)], the resulting sign can always be absorbed by the phase of the holon condensate ( $\oint d\mathbf{r} \cdot \nabla\phi_h = \pm 2\pi$ ), without changing  $\oint_c d\mathbf{r} \cdot \mathbf{A}^e$  in (14). To paraphrase, each magnetic flux can trap either an  $S_z = +1/2$  or an  $S_z = -1/2$  spinon *without* changing the minimal quantization  $hc/2e$ . However, this two-fold degeneracy will be split by a Zeeman coupling to the magnetic field. This leads to a very unique consequence of the theory, discussed below.

The Zeeman splitting between the  $S_z = \pm 1/2$  inside a vortex core can give rise to a Schottky contribution to the low temperature specific heat. From our analysis, we expect the number of magnetic moments [Fig. 4(b)] to be directly proportional to the applied magnetic field  $H$ , at least for small fields. The constant of proportionality can be estimated easily. Assuming a lattice constant of  $4.4\text{\AA}$  for the Cu-O planes, we find the number of induced magnetic moments given by

$$n_{\text{moment}} \simeq \eta \times H,$$

where  $\eta = 8 \times 10^{-5}$  per Cu Tesla. Therefore, for a field of  $6T$ , we estimate the number of induced magnetic moments to be  $\sim 0.05\%$  per Cu. Evidence for magnetic moments can be seen in the measurements of field dependent specific heat at low temperatures. However, most of these measurements are done on YBCO and the moments are invariably ascribed to impurities associated with the chains. Here, we offer another explanation based on our theory. The low temperature specific heat of the 123, 124 and 247 phases of YBCO has been measured by Sanchez *et al.* [17]. They find that the observed specific heat in all these compounds can be described satisfactorily if one were to include a magnetic Schottky contribution whose amplitude (*i.e.*, the number of magnetic moments) increases linearly with the field. For an applied field of  $6T$ , they estimate the number of induced magnetic moments in all three phases of YBCO to be  $0.07 - 0.08\%$  per Cu atom. Similar results are observed in LSCO [18], where the number of induced magnetic moments is also

found to increase linearly for small fields (up to  $7T$ ) before saturating. Mason *et al.*, [18] estimate the number of induced magnetic moments to be around  $0.07\%$  per Cu at  $6T$ . These observations are in agreement with our estimates for the number of induced magnetic moments, as well as the field dependence. In another study of field dependent specific heat in YBCO, Emerson *et al.*, [19] determined the number of induced magnetic moments to be  $0.05 - 0.1\%$  per Cu in the same range of magnetic fields. But they also find that the number of magnetic moments does not vary with the applied field. However, as the authors point out, their data is inherently different from the data of Sanchez *et al.* This discrepancy needs to be resolved by future measurements. Recently, Moler *et al.*, [20] measured the field dependent specific heat of YBCO and also observed the number of magnetic moments to be  $\sim 0.1\%$  per Cu.

Finally, we comment on the recent specific heat measurements in Zn doped YBCO [21]. These measurements show that the free magnetic moments (induced by Zn substitution) that are present in the normal state do not contribute to the specific heat below  $T_c$ . There is no Schottky anomaly due to the magnetic moments for a wide range of Zn concentration in fields up to  $8T$ . The authors explain this in terms of Kondo screening in the superconducting phase. However, the present work may provide a different explanation based on the confinement of bosonic spinons in the superconducting state. Except for the free moments trapped at vortex cores, no local moments related to  $S = 1/2$  bosonic spinons are allowed in the superconducting bulk. This explains why the Schottky anomaly remains unchanged as in the Zn-free case up to  $1\%$  Zn concentration.

*Enhanced AF correlations:* Thus far, we have seen that a spinon is trapped inside the core of a vortex. Clearly, the amplitude of the holon condensate  $\rho_h$  vanishes at the trapped spinon position according to the GL equation (7), consistent with the single occupancy constraint. Since the spinon has a characteristic size  $a_c$ , we expect the holon density to be reduced within such a core region. This, in turn, enhances the RVB pairing inside the core. Consider the Hamiltonian (4) for the spinons. At finite doping, the AF correlations are suppressed by the vortex effect of the *holon* link field  $A_{ij}^h$ . Now, if the holon density is reduced inside the core region, then there is a reduction in the effect of  $A_{ij}^h$  inside the core, and consequently, RVB pairing or equivalently, the short range AF correlations are *enhanced* compared to the bulk. In the extreme case of  $A_{ij}^h \rightarrow 0$ , when there are no holon vortices, the correlations inside the core will mimic those of the undoped compound. Thus, when a spinon is trapped near the vortex core, there is a concomitant enhancement of AF correlations inside the core region. This is a very important consequence of our theory and should be contrasted with the results from slave-boson RVB theories. In the latter, *both* the holon condensate and the

RVB pairing are *suppressed* inside the core region. Consequently, AF correlations are absent in the core region which is expected to be a normal Fermi liquid [14], [15]. On the other hand, our considerations lead to the conclusion that the core region is closer to the underdoped regime with *enhanced* AF correlations.

Based on these arguments, we can obtain some simple estimates. Recall that in the bulk, the characteristic length scale of a spinon,  $a_c = a/\sqrt{\pi\delta}$ . Within the magnetic vortex core, due to the reduction of  $\delta$ , we expect the core size to increase, and the new core size,  $l_c = \sqrt{\delta/\delta_c}a_c$ , where  $\delta_c$  denotes the average holon density within the core. To be consistent with the condition that one holon is expelled from the core region due to the trapping of an extra spinon, we demand  $\rho_c \times \pi l_c^2 = \rho \times \pi l_c^2 - 1$ , where  $\rho_c = \delta_c/a^2$  and  $\rho = \delta/a^2$ . We then get  $\delta_c = \delta/2$ . Thus, the holon density is reduced by half within the core and  $l_c = \sqrt{2}a_c$ . Since  $E_g \propto \delta$  in the bulk, we conclude that the characteristic spin excitation energy should be also approximately reduced by half, *viz.*,  $E_g^{core} \simeq E_g/2$ . Recently, Lake *et al.*, reported the observation of AF correlations by neutron scattering inside the vortex core [22], where a field-induced sub-gap spin excitation was found in LSCO at an energy scale  $\sim 4.3$  meV, approximately half of the characteristic spin energy scale in the bulk. The reduction of  $E_g^{core}$  due to the reduced holon density may provide an explanation for such an observation. Tunneling spectroscopy of the vortex cores shows that the quasiparticle gap inside the core region is larger compared to the magnitude of the bulk superconducting gap [23]. Again, this result does not contradict our conclusion that the core region is closer to the underdoped regime. These issues will be discussed in detail elsewhere [10].

To conclude, we presented an effective theory of the superconducting state based on the bosonic representation of the  $t-J$  model. The theory leads to several interesting consequences. We first showed that the bosonic RVB theory leads to phase vortices in the superconducting order parameter. The phase vortices are excited spinons and  $T_c$  is the temperature at which the cores of these vortices begin to overlap. We showed how  $hc/2e$  flux quantization leads to the trapping of a spinon inside the vortex core and argued that the cores exhibit enhanced anti-ferromagnetic correlations. Our estimates for the core energy show that the bosonic RVB theory does not allow for  $hc/e$  flux quanta for any doping concentration. We also showed that the trapping of a spinon inside the vortex core leads to observable consequences for the low temperature specific heat. In a forthcoming publication, we shall present a quantitative analysis of the structure of an isolated vortex, based on the ideas outlined in this paper. We believe that the approach presented in this paper can bridge the gap between a microscopic model such as the  $t-J$  Hamiltonian and effective theories of quasiparticles in a  $d$ -wave superconductor coupled to fluctuating vortices [24].

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Fig. 1. Holons and spinons perceive each other as carrying fictitious  $\pi$  flux tubes.

Fig. 2. (a) An  $S = 1/2$  bosonic spinon carries a current vortex in the superconducting phase; (b) An  $S = 1$  spin excitation is constructed from a pair of confined  $S = 1/2$  spinon vortices.

Fig. 3. Phase coherence disappears at  $T_c$ , as spinon vortices unbind due to core touching;  $T_c$  is determined by the characteristic energy,  $E_g$ , of the  $S = 1$  excitation. The experimental data are from Ref. [11].

Fig. 4. Flux quantization occurs at  $hc/2e$ , with a bosonic  $S = 1/2$  spinon trapped inside the core.

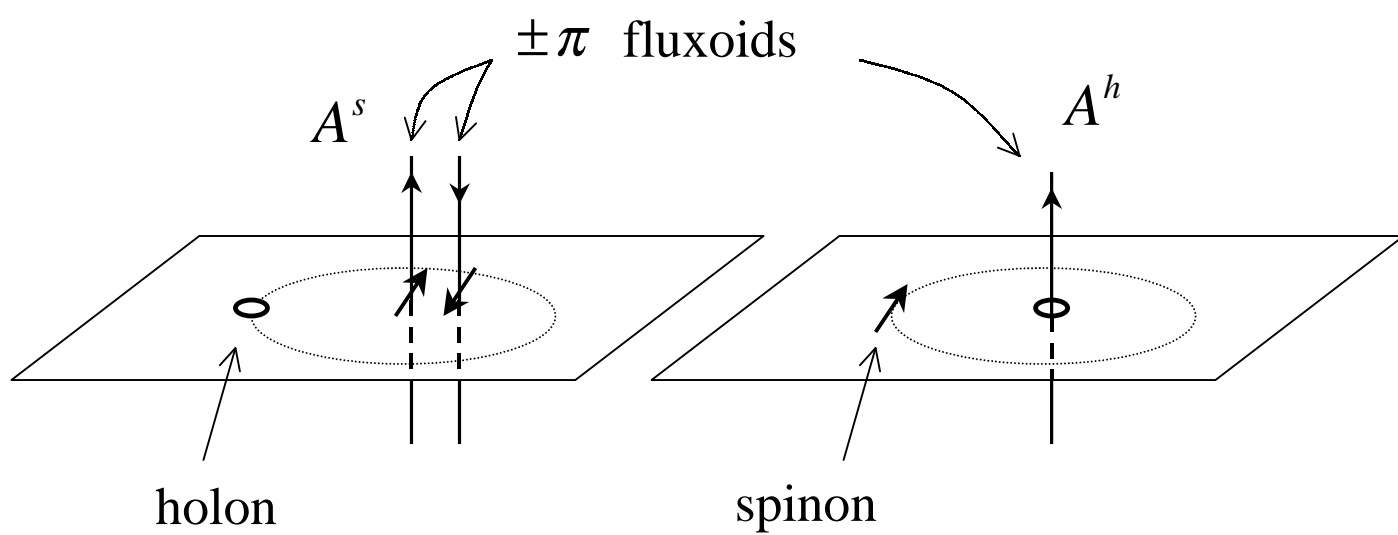


Fig. 1

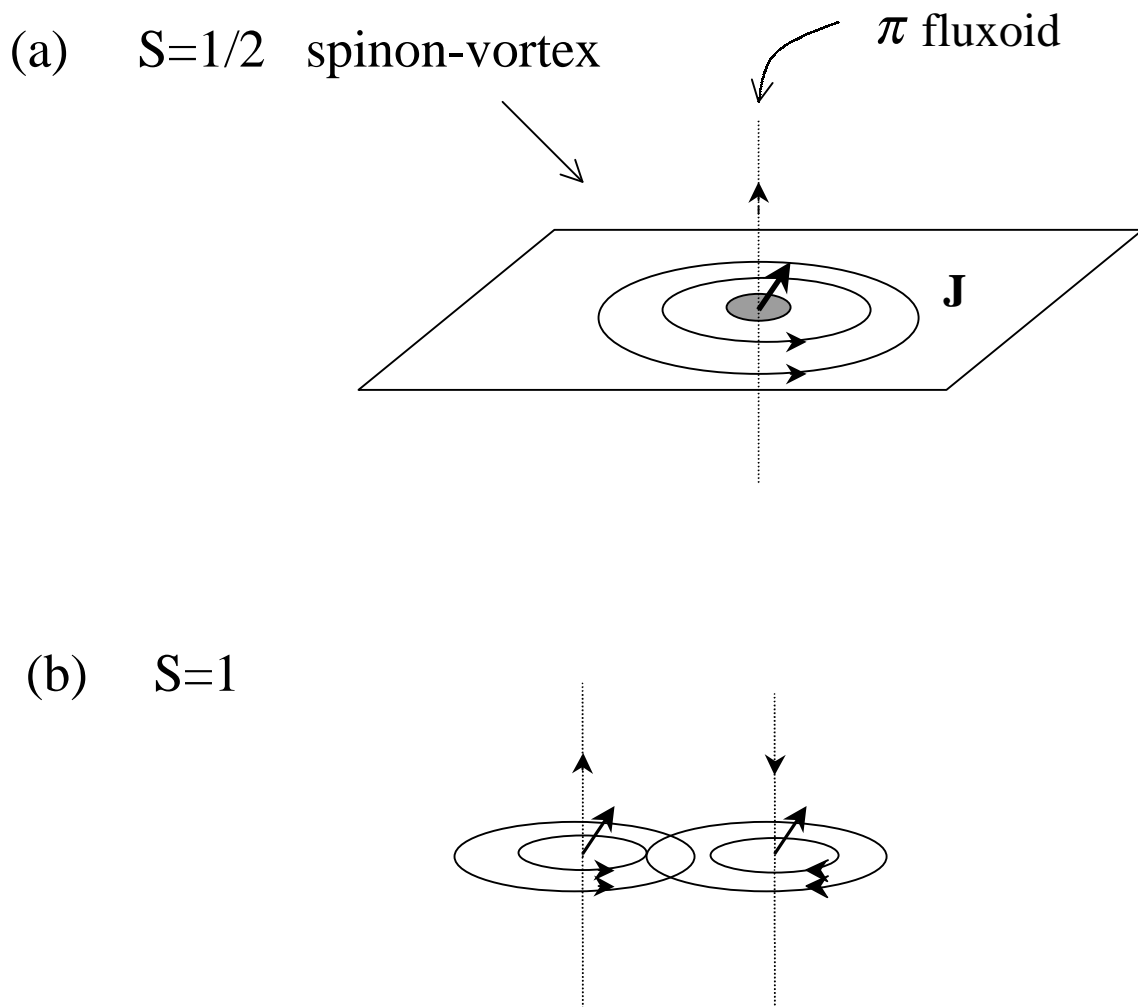
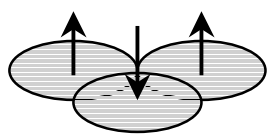
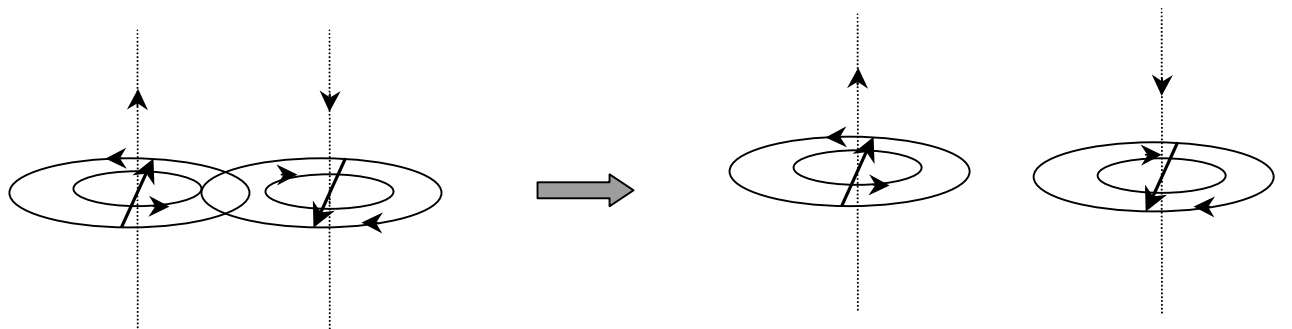


Fig. 2





vortex core touching

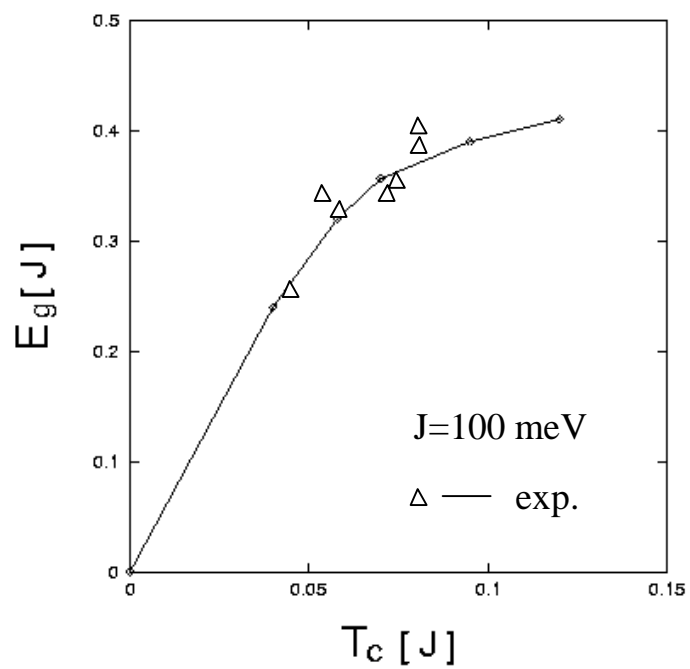
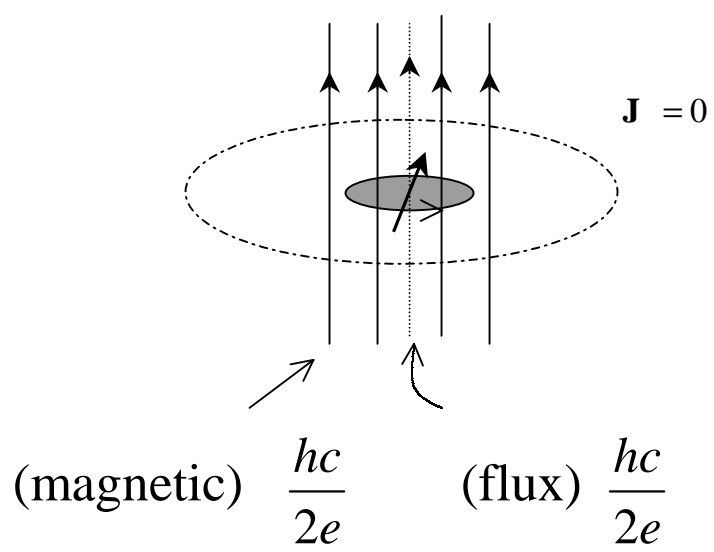


Fig. 3

(a)



(b)

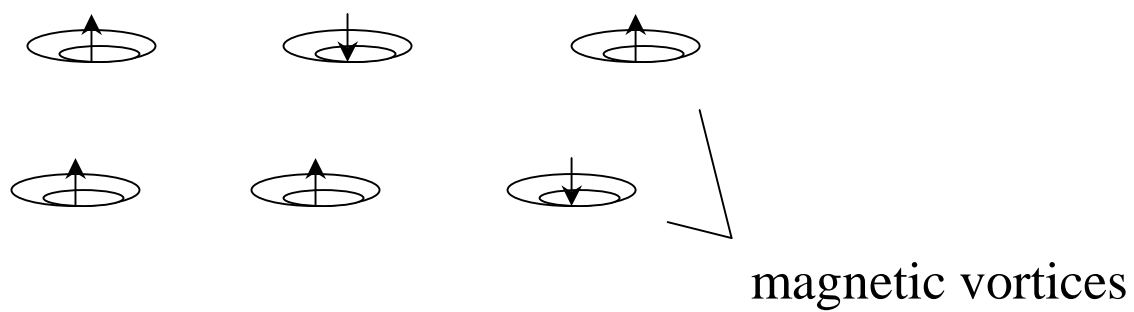


Fig. 4